CHAPTER 30 Faraday's Law

Answers to Understanding the Concepts Questions

1. By Gauss' law for magnetism, the magnetic flux through a closed surface is always zero. Thus there will be no induced electric field. Another way of seeing this is to view the sphere as consisting of two adjacent hemispheres separated by the equator. Suppose for example that $\vec{B}$ is oriented perpendicular to the plane of the equator and increases with time. The normal to the surface of the lower atmosphere points in a direction opposite to that of $\vec{B}$, hence by Faraday's law there is an induced electric field on the equatorial boundary of the lower hemisphere due to a decreasing $\vec{B} \cdot d\vec{A}$. On the upper hemisphere the normal to the surface is aligned with the direction of B, there is an increasing $\vec{B} \cdot d\vec{A}$, and the induced electric field on the equatorial boundary of the upper hemisphere win be the san1e as that of the lower atmosphere, but of opposite sign. The two contributions cancel.

2. No. An electric field is induced in an imaginary loop in vacuum whenever the magnetic flux through the surface enclosed by the loop varies with time. If a real conducting loop is present, the induced electric field can drive an induced current in the loop.

3. (a) is not necessarily correct; it depends on whether the N-pole or the S-pole is brought closer to the loop. (b) is incorrect; Lenz's law does apply. (c) is incorrect, with an induced emf there is actually an induced current in the wire loop. (d) is also incorrect; the induced emf depends only on the rate of change of the magnetic flux through the loop, but not on its resistance. (e) is the correct choice. Note that the current in the wire loop satisfies Ohm's law.

4. By twisting the wires together that might be part of a closed circuit, the effects of Faraday's law are drastically reduced. You can contrast this case with a case in which the wires are widely separated: this opens a large area and hence the possibility for a large magnetic flux. Pulling the wires together reduces this possibility; twisting them reduces it even further, because then there are a series of very small areas, and successive areas are oppositely oriented, reducing the possibility of a flux even further.

5. Yes. The magnetic flux is $\Phi_B = \iint \vec{B} \cdot d\vec{A}$, and there are many ways to change $\vec{B}$ without changing Φ_B. Here are just two examples: (a) Divide the region into two subregions of equal areas, increase $\vec{B}$ in the first region at a certain rate and decrease it in the second one at the same rate. The changes in Φ_B of the two subregions cancel out. (b) Introduce a variable component of $\vec{B}$ that is perpendicular to $d\vec{A}$. Regardless of how this component changes, its contribution to $\vec{B} \cdot d\vec{A}$ is always zero, so Φ_B does not change.

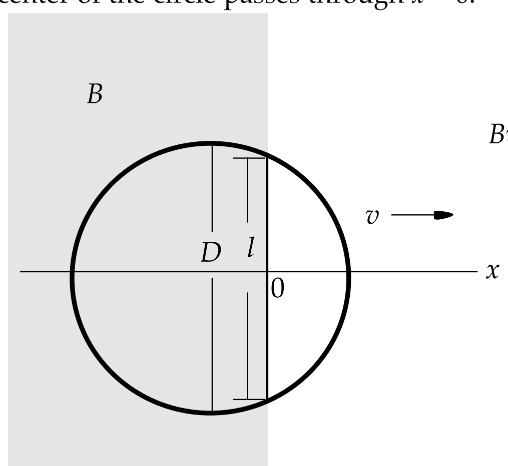
6. As the north pole of the bar magnet approaches the wire loop, the flux through the loop increases over time (if we choose down as the positive direction), so the induced current is counterclockwise (as that would produce a counter-acting flux in accordance with Lenz's law). Later on, as the south pole of bar magnet leaves the loop, the flux through the loop decreases, so the induced current runs backwards, i.e., it becomes clockwise. The correct choice is therefore (b).

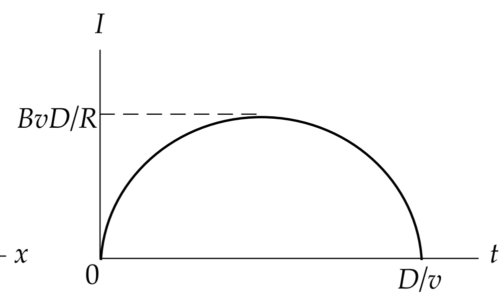
7. (a) No: the flux entering from the right increases, and by Lenz' law the current must be such as to counteract that. (b) Yes: the induced current gives rise to a field that opposes the increase in the flux due to the approaching loop. (c) No: when the switch is closed, there is a rising flux in the direction to the right. The induced current must counter that. (d) No: the loop is oriented so that there is no magnetic flux through it from the straight wire, hence no current is induced.

8. There is no change in the magnetic flux through the hoop as it rolls, since the area of the hoop facing the terrestrial magnetic field lines as well as the strength of the field itself are both constant. So no induced emf or current will result.

9. As long as the loop is in a region in which the magnetic field is uniform, and it does not change its orientation, then the magnetic flux is constant and there is no induced emf. If the loop enters or leaves the region of constant field, then there win be a change in flux, and there win be an emf, unless the plane of the loop is parallel to the direction of the magnetic field vector.

10. See the sketch below. The emf in the loop is given by $\mathcal{E} = Blv$, and the current is $I = \mathcal{E}/R = Blv/R$, with R the resistance of the loop. Note that l reaches its maximum value of D, the diameter of the loop, when the center of the circle passes through $x = 0$.





11. Yes it does. Pulling the sheet of metal out of the region between the two poles changes the magnetic flux through the sheet from a finite value to zero. The change in magnetic flux results in an induced emf, which drives a current in the sheet. As the current flows it generates thermal energy, which must come from the positive work done in pulling the sheet out.

12. In each case, as the magnet is thrust into the tube, the magnetic flux through the cross-section of the tube changes, and an emf is induced. In the case of a copper tube this induced emf drives a relatively large current through the tube, and as a result there is relatively strong a magnetic resistance on the falling magnet, which as a result falls slower than the case of free fall. The only difference for an aluminum tube (of the same dimensions, we assume) is that its resistance is higher than that of the copper tube, so the induced current is not as strong, and the falling motion of the magnetic is not affected by it as strongly as in the case of the copper tube. Since plastic is an insulator, no current is generated even though there is still an induced emf. Therefore the plastic tube provides no magnetic resistance to the falling motion of the magnet.

13. There will be no change in any physically measurable quantity. The reason is the invariance of the laws of electromagnetism under Galilean transformations: the laws of motion are the same in all inertial frames.

14. As the plate oscillates in and out of the region in between the two magnetic poles the magnetic flux through its surface changes. An induced emf result in the plate. If the plate is metallic then this induced emf drive a current, which generates thermal energy — taken from the mechanical energy of the oscillation of the plate. Eventually as all the mechanical energy of the plate is depleted it stops. One way to prevent this from happening is to use an insulating material, instead of metal, for the plate. That way no current is induced in the plate.

15. As the water is dammed up it is at a higher level. Once it flows down it gives up its gravitational potential energy in exchange for kinetic energy. The fast-flowing water at the bottom of the dam is used to drive the blades of a large turbine. As the turbine blades are forced to rotate inside a magnetic field (which can be provided by permanent magnets) an emf is induced, which is then distributed through the power transmission lines.

16. Consider, for definiteness, a flow of plasma directed from the top to the bottom of this page, up in the plane of the paper, and a magnetic field directed into the plane of the paper. The force on moving positive charges win be to the left. As a result, these charges will be deposited on the left-hand boundary, with a consequent an electric field to the right. This electric field will increase as charges accumulate until it is strong enough to prevent more charges from accumulating. This happens when $\vec{E} + \vec{v} \times \vec{B} = 0$. Thus $v = E/B$, independent of the density of the carriers and the magnitude of their charge. The same analysis leads to the same speed if the carriers are negative.

17. This is similar to the situation discussed in Question 12 above. An induced current is generated in the copper tube as the magnet enters the tube, since the magnetic flux through the tube is changing. Since the tube is long, however, once the magnet has fallen deep into the tube the magnetic flux through the tube no longer changes, and as a result the induced emf and current drop to zero.

18. In any case there is an induced emf on the straight wire. If the wire is part of a closed circuit with finite electrical resistance, then this induced emf would result in a current. If, instead, the wire is isolated and open at both ends, then no current can flow in it.

19. When the circuit is closed, there is a change in the current in the wire, and therefore a change of magnetic flux in the iron core. This induces an emf in the aluminum ring, and the current generated by this emf must be in a direction opposite to that in the coil. Since two antiparallel currents repel, the aluminum ring jumps. The repulsion can be thought of as a direct manifestation of Lenz' law.

20. (a): The magnetic field produced by the current in the solenoid changes as the voltage that drives the current varies. This results in a change in the magnetic flux through the ring. An emf is then induced in the ring, and a current results. As the current flows through the ring it generates Joule heat ($P = I^2R$), so the ring gets hot. (b) and (c): The thermal energy generated in the ring is the result of an induced current, which is due to the varying magnetic flux created by the variable voltage that drives the current in the solenoid. The thermal energy in the ring ultimately comes from the voltage source, i. e., it is converted from the electrical energy needed to drive the current through the solenoid.

21. A harmonically varying voltage across the solenoid gives rise to a harmonically varying current through the solenoid, and therefore a harmonically varying magnetic flux. This induces an emf in the copper ring, which in its turn gives rise to a current flowing in the copper. The current in the copper ring will always be opposite in direction to the current in the coil by Lenz' law. Since opposite currents repel, the copper ring has a tendency to be accelerated in a direction opposite to the direction of gravity. The copper ring must be of such a radius and thickness that the two forces cancel.

22. We can fix a pair opposite magnetic poles in between the metal spokes of a wheel. As the wheel turns, the spokes cut through the magnetic field lines, generating an induced emf which can be used to power the light.

23. Most cars have iron frames with some residual magnetic fields. As a car drives over the embedded wire loop it changes the magnetic flux through the loop, and that causes an induced current in the loop. This current can be used to change the signal.

24. A positive sign in front of the time rate of change of magnetic flux for the induced emf means that, once the current in an inductor increases (thereby increasing the magnetic flux through it), the induced emf is in the same direction of the current, which in turn increases even further as a result — we would in fact be able to generate "free" electrical power, using by using an inductor. This violates the conservation of energy. (Sadly, if you continue to leave a lightbulb hooked up to a battery pack after you turn it on, then the inductance of the lightbulb would drive the current higher and higher, and the bulb glows brighter and brighter, only it does not last very long — it invariably overheats and burns out!)

Solutions to Problems

1. Connect the coil to the bulb and place the coil between the poles of the horseshoe magnet. Rotate the magnet around the coil (which may be easier than rotating the coil). To make the bulb brighter increase the speed of rotation, which increases the rate at which the magnetic flux changes.

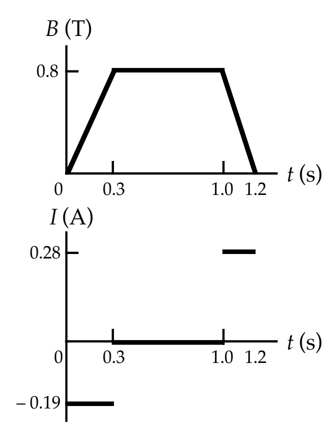
2. (a) The magnetic flux through the galvanometer coil from the bar magnet is to the right and increasing. To oppose this <u>increase</u>, the flux created by the induced current must be to the left, which requires the induced current shown.
   (b) When the switch is closed, the magnetic flux through the galvanometer coil from the top coil is down and increasing. To oppose this <u>increase</u>, the flux created by the induced current must be up, which requires the induced current shown.
   (c) When the galvanometer coil moves up, the magnetic flux through the galvanometer coil from the top coil is down and increasing. To oppose this <u>increase</u>, the flux created by the induced current must be up, which requires the induced current shown.
   (d) When the galvanometer coil swings into the magnetic field of the top coil, the component of the area vector parallel to the magnetic field increases and thus the downward magnetic flux from the top coil is increasing. To oppose this <u>increase</u>, the flux created by the induced current must be up, which requires the induced current shown.

3. The magnetic flux through the loop is
   $$\Phi_B = \iint \vec{B} \cdot d\vec{A} = BA.$$
   The magnitude of the induced emf is
   $$\mathcal{E} = d\Phi_B/dt = A\, dB/dt = A(B_2 - B_1)/\Delta t$$
   $$= (12 \times 10^{-4}\,\text{m}^2)(2.0\text{ T} - 1.5\text{ T})/(5.7\text{ s}) = \boxed{1.05 \times 10^{-4}\text{ V}}.$$

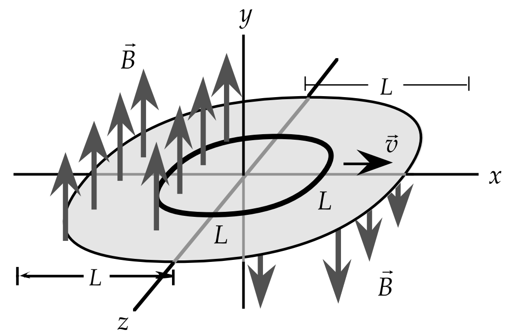
4. The current in the loop is
   $$I = \mathcal{E}/R.$$
   Because the emf is constant, we have
   $$P = I^2R = \mathcal{E}^2/R = (1.05 \times 10^{-4}\text{ V})^2/(7.7\ \Omega) = \boxed{1.4 \times 10^{-9}\text{ W}}.$$

5. The magnetic flux through the loop is
   $$\Phi_B = \iint \vec{B} \cdot d\vec{A} = \iint (B\hat{i}) \cdot (dA\,\hat{i}) = BA.$$
   The magnitude of the current in the loop is
   $$I = \mathcal{E}/R = (d\Phi_B/d\tau)/R = (A/R)\, dB/dt = (\pi r^2/R)\, dB/dt;$$
   $$2\text{ A} = [\pi(3.5 \times 10^{-2}\text{ m})^2/(1.5 \times 10^{-3}\ \Omega)]\, dB/dt, \text{ which gives } d\vec{B}/dt = \boxed{(0.78\text{ T/s})\hat{i}}.$$
   Since the direction of current is not specified, so the magnetic field could be increasing or decreasing.

6. The current in the ring is
   $$I = \mathcal{E}/R = -(d\Phi_B/dt)/R = -(A/R)\, dB/dt = -(A/R)\, \Delta B/\Delta t.$$
   For the three segments we have
   $$0 < t < 0.3\text{ s:}\quad I = -(A/R)\,\Delta B/\Delta t = -[(14 \times 10^{-4}\text{m}^2)/(0.02\ \Omega)][(0.8\text{ T})/(0.3\text{ s})] = -0.19\text{ A}.$$
   $$0.3\text{ s} < t < 1.0\text{ s:}\quad I = -(A/R)\,\Delta B/\Delta t = -[(14 \times 10^{-4}\text{m}^2)/(0.02\ \Omega)][(0)/(0.7\text{ s})] = 0.$$
   $$1.0\text{ s} < t < 1.2\text{ s:}\quad I = -(A/R)\,\Delta B/\Delta t = -[(14 \times 10^{-4}\text{m}^2)/(0.02\ \Omega)][(-0.8\text{ T})/(0.2\text{ s})] = +0.28\text{ A}.$$



$\boxed{7}$. We take $t = 0$ when the leading edge of the loop enters the magnetic field. The position of this edge is given by $x = -L + vt$. The leading edge will reach the z-axis at $t_1 = L/v$ and the other side at $t_2 = 2L/v$. The trailing edge will leave the magnetic field at $t_3 = 3L/v$.



We have three regions to consider:

Region I, $0 < t < (L/v)$.

The magnetic flux through the loop is

$\Phi_B = BL(x + L)$. (Note that $x < 0$.)

The induced emf is

$$\begin{aligned}\mathcal{E}_1 &= -\,d\Phi_B/dt \\ &= -\,BL(dx/dt) = \boxed{-BLv \text{ (clockwise as viewed from above)}}.\end{aligned}$$

Region II, $(L/v) < t < (2L/v)$.

The magnetic flux through the loop is

$\Phi_B = [+\,B(L - x) - B(x)]L = BL(L - 2x)$. (Note that $x > 0$.)

The induced emf is

$$\begin{aligned}\mathcal{E}_2 &= -\,d\Phi_B/dt \\ &= -\,BL(-\,2\,dx/dt) = \boxed{+2BLv \text{ (counterclockwise)}}.\end{aligned}$$

Region III, $(2L/v) < t < (3L/v)$.

The magnetic flux through the loop is

$\Phi_B = -\,B(2L - x)L = B(x - 2L)L$. (Note that $x > L$.)

The induced emf is

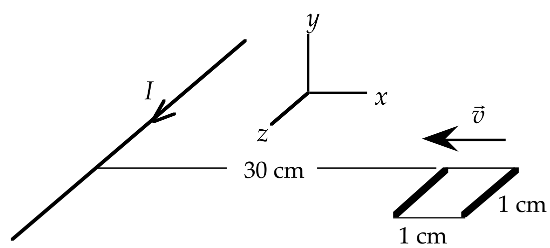
$$\begin{aligned}\mathcal{E}_3 &= -\,d\Phi_B/dt \\ &= -\,BL(dx/dt) = \boxed{-BLv \text{ (clockwise)}}.\end{aligned}$$

8. With the current in the $+\,z$-direction, the magnetic field of the wire is up in the region where the loop is located:

$B = +\,\mu_0 I/2\pi x$,

so the flux through the loop is

$\Phi_B = BA = +\,(\mu_0 I/2\pi x)A$.



Because the motion is uniform and we ignore the variation of the magnetic field across the loop, we find the emf induced in the loop from

$$\begin{aligned}\mathcal{E} &= -\,\Delta\Phi_B/\Delta t = -\,A(B_f - B_i)/\Delta t \\ &= -\,A(\mu_0 I/2\pi)(1/x_f - 1/x_i)/\Delta t \\ &= -\,[(2.5\times10^{-2}\ \text{m})^2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.120\ \text{A})/2\pi][1/(0.14\ \text{m}) - 1/(0.15\text{m})]/(0.05\ \text{s}) \\ |\mathcal{E}| &= \boxed{1.3\times10^{-10}\ \text{V clockwise}}.\end{aligned}$$

$\boxed{9}$. If we take the area of the coil to be aligned with the field at t = 0, we have

$\Phi_B = N\vec{B}\cdot\vec{A} = NBA\cos(\omega t) = NBA\cos(2\pi f t)$.

The emf induced in the coil is

$$\begin{aligned}\mathcal{E} &= -\,d\Phi_B/dt = -\,N\,(d/dt)[BA\cos(2\pi f t)] \\ &= NBA2\pi f\,\sin(2\pi f t).\end{aligned}$$

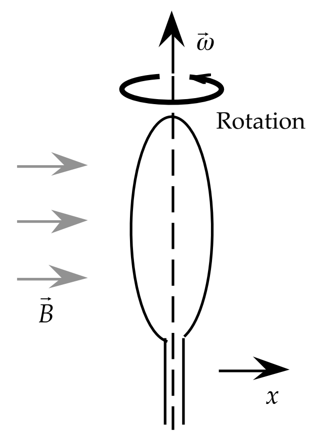
The maximum value occurs when $\sin(2\pi f t)$ is maximum:

$\mathcal{E}_{\max} = NBA2\pi f\,(1) = (100\ \text{turns})(0.30\ \text{T})(8.0\times10^{-2}\ \text{m})^2 2\pi(15\ \text{Hz}) = \boxed{18\text{V}}$.

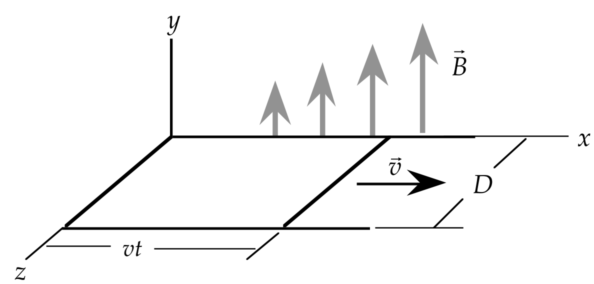
10. If we take the area of the coil to be aligned with the field at $t = 0$, we have
$\Phi_B = N\vec{B} \cdot \vec{A} = NBA\cos(\omega t) = NBA\cos(2\pi ft)$.
The emf induced in the coil is
$\mathcal{E} = -\,d\Phi_B/dt = -\,N\,(d/dt)[BA\cos(2\pi ft)]$
$= NBA2\pi f\,\sin(2\pi ft)$.
The current in the coil is
$I = \mathcal{E}/R = NBA2\pi f\,\sin(2\pi ft)/R$.
The maximum value occurs when $\sin(2\pi ft)$ is maximum:
$I_{max} = NBA2\pi f/R$
$3.0\text{ A} = (450\text{ turns})(0.35\text{ T})\pi(2.5 \times 10^{-2}\text{ m})^2 2\pi f/(12\ \Omega)$, which gives $f = \boxed{19\text{ Hz }(1.6 \times 10^2\text{ rad/s})}$.

11. We take $t = 0$ when the leading corner of the loop passes the origin and enters the magnetic field. Before $t = 0$, there is no flux through the coil, and thus there is no emf. The trailing corner will enter the magnetic field at $t = L/v_0$. After this time the flux through the coil will be constant, and thus there is no emf. Between these two times, each side of the coil will have moved a distance v_0t into the magnetic field. The area of the coil through which there is a magnetic field is
$\vec{A} = (v_0t)^2\hat{k}$.
The magnetic flux through the coil is
$\Phi_B = \vec{B} \cdot \vec{A} = [B_0(\hat{i} + \hat{j} + \hat{k})] \cdot [(v_0t)^2\hat{k}] = B_0v_0^2t^2$.
The induced emf is
$\mathcal{E} = -\,d\Phi_B/dt = \boxed{-2B_0v_0^2t}$, $0 < t < L/v_0$.

12. At $t = 0$, the flux through the loop is maximum. The rate of change of the flux, however, is zero, so $\mathcal{E} = \boxed{0}$.
At $t = T/4$, the flux through the loop is zero. The flux is changing from entering one side of the loop to entering the other side. The induced current will oppose this change by producing a magnetic field that will maintain the original flux. For the view in the figure, the induced field is directed into the page, and $\mathcal{E}$ is $\boxed{\text{clockwise}}$.
At $t = T/2$, the flux through the loop is maximum. The rate of change of the flux, however, is zero, so $\mathcal{E} = \boxed{0}$.
At $t = 3T/4$, we have the same view as for $t = T/4$. $\mathcal{E}$ is $\boxed{\text{clockwise}}$.
Note that, in the reference frame of the loop, the emf has reversed.



13. Because the magnetic field varies in the x-direction but not the y-direction, we find the flux through the loop at time t by choosing a strip with length D and thickness dx:
$$\Phi_B = \iint \vec{B} \cdot d\vec{A} = \int_0^{vt} (Cx\hat{j}) \cdot (D\,dx\,\hat{j}) = \tfrac{1}{2}CDvt^2.$$
The induced emf is
$\mathcal{E} = -\,d\Phi_B/dt = -\,CDv^2t$ clockwise.
The resistance of the loop is $R = \alpha L = \alpha(2D + 2vt)$.
The current in the loop is
$I = \mathcal{E}/R = \boxed{-CDv^2t\,/2\alpha(D + vt)\ \text{clockwise}}$.
This differs from the result of Example 31–5 by the presence of a vt term in the numerator. Here the cause of the emf is both the increasing area and the increasing magnetic field.



14. The magnetic flux through the loop is

$$\Phi_B = \vec{B} \cdot \vec{A} = (B_0\hat{k}) \cdot (Dvt\hat{j}) = 0.$$

Thus there is no flux and no change in flux through the loop. There is no induced emf; the induced current is $\boxed{0}$.

15. Because the magnetic field varies, we choose a differential area dx dz $\hat{j}$ and integrate to find the flux through the loop:

$$\Phi_B = \iint \vec{B} \cdot d\vec{A} = \iint (Cz\hat{j}) \cdot (dx\, dz\,\hat{j})$$

$$= \iint Cz\, dx\, dz = \int_0^D Cz\, dz \int_0^{vt} dx = \tfrac{1}{2}CD^2vt\ .$$

The induced emf is

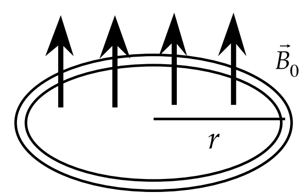
$$\mathcal{E} = -\,d\Phi_B/dt = -\tfrac{1}{2}CD^2v.$$

The resistance of the loop is $R = \alpha L = \alpha(2D + 2vt)$. The current in the loop is

$$I = \mathcal{E}/R = \boxed{-CD^2v/4\alpha(D + vt)\ \text{clockwise}}.$$

16. From the expression for the radius, $r = r_0(1 + \alpha t)$, we have

$$dr/dt = r_0\alpha.$$



Because the magnetic field is constant and perpendicular to the ring, we find the flux through the loop from

$$\Phi_B = \vec{B} \cdot \vec{A} = B_0A = B_0\pi r^2.$$

The induced emf is

$$\mathcal{E} = -\,d\Phi_B/dt = -\,2B_0\pi r\,(dr/dt) = -\,2B_0\pi[r_0(1 + \alpha t)]r_0\alpha\ \ \text{clockwise}.$$

The resistance of the ring is

$$R' = R2\pi r = R_0(1 + \beta t)2\pi r_0(1 + \alpha t),$$

so the induced current is

$$I = \mathcal{E}/R' = -\,2B_0\pi r_0{}^2\alpha(1 + \alpha t)/[R_0(1 + \beta t)2\pi r_0(1 + \alpha t)]$$

$$= \boxed{-B_0r_0\alpha/R_0(1 + \beta t)\ \ \text{clockwise}}.$$

17. If we take the area of the coil to be aligned with the field at $t = 0$, we have

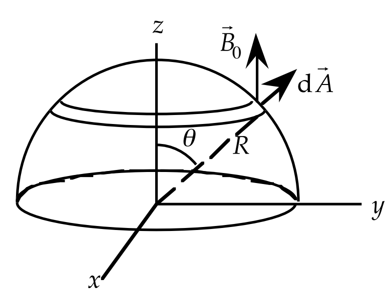
$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos(\omega t).$$

The emf induced in the coil is

$$\mathcal{E} = -\,d\Phi_B/dt = -\,(d/dt)[BA\cos(\omega t)] = \boxed{BA\omega\ \sin(\omega t)}.$$

18. Because the angle between the magnetic field and the area varies over the surface of the hemisphere, we find the flux by integration. We choose a strip at an angle θ with a thickness R dθ, as shown in the diagram. The area of this strip is

$$dA = (2\pi R \sin\theta)R\, d\theta = 2\pi R^2 \sin\theta\, d\theta.$$



From the diagram, we see that θ is the angle between $\vec{B}$ and d$\vec{A}$, so we have

$$\Phi_B = \iint \vec{B} \cdot d\vec{A} = \int_0^{\pi/2} B_0\,(\cos\theta)\,2\pi R^2 \sin\theta\, d\theta$$

$$= B_0 2\pi R^2\left(\frac{\sin^2\theta}{2}\right)\Bigg|_0^{\pi/2} = B_0\pi R^2.$$

This is the flux of a constant field through the area of a circle of radius R, as found in Example 30–3.

19. Because the velocity is perpendicular to the magnetic field and the antenna, we find the magnitude of the emf from

$$\mathcal{E} = BLv$$
$$= (2 \times 10^{-10}\text{ T})(5\text{ m})(8 \times 10^{3}\text{ m/s}) = \boxed{8 \times 10^{-6}\text{ V}}.$$

20. The component of the magnetic field perpendicular to both the velocity and the wing produces the magnetic flux through the area swept out by the wing, so we find the magnitude of the emf from

$$\mathcal{E} = B_{\perp}Lv = (2.8 \times 10^{-5}\text{ T})(35\text{ m})(940\text{ km/h})(1\text{ h}/3.6\text{ ks}) = \boxed{0.26\text{ V}}.$$

If the velocity is to the east, the component of the magnetic field perpendicular to both the velocity and the wings is the same, so the answer would not change.

21. Because the velocity is perpendicular to the magnetic field and the rod, we find the magnitude of the potential difference from

$$\mathcal{E} = BLv$$
$$= (0.069\text{ T})(0.25\text{ m})(1.7 \times 10^{-2}\text{ m/s}) = \boxed{2.9 \times 10^{-4}\text{ V}}.$$

22. Because the velocity is perpendicular to the magnetic field and the rod, we find the magnitude of the potential difference when the speed is v from

$$\mathcal{E} = BLv.$$

The rate at which this potential difference changes is

$$d\mathcal{E}/dt = BL\,dv/dt = BLg = (1.5 \times 10^{-4}\text{ T})(0.06\text{ m})(9.8\text{ m/s}^2) = \boxed{8.8 \times 10^{-5}\text{ V/s}}.$$

23. We find the magnetic field from

$$\mathcal{E} = \tfrac{1}{2}B_0\omega L^2\,;$$
$$100 \times 10^{-3}\text{ V} = \tfrac{1}{2}B_0(100\text{ rad/s})(0.10\text{ m})^2, \text{ which gives } B_0 = \boxed{0.20\text{ T}}.$$

24. We find the flux through the area swept out by a radial line on the disk, as in Example 30–7:

$$\Phi_B = B(\tfrac{1}{2}\theta R^2) = \tfrac{1}{2}B\omega t R^2.$$

The induced voltage is

$$\mathcal{E} = -\,d\Phi_B/dt = -\tfrac{1}{2}B\omega R^2$$
$$= -\tfrac{1}{2}(0.10\text{ T})(150\text{ rad/s})(0.035\text{ m})^2 = \boxed{-9.2 \times 10^{-3}\text{ V}}.$$

The negative sign indicates that when the direction of $\vec{\omega}$ is the same as the direction of $\vec{B}$, the emf is directed radially out from the axis, and the rim is at the higher potential.

$\boxed{25}$. When the bar has moved a distance x, the area swept out by the bar is

$$A = xL\sin 60°,$$

so the magnetic flux through the area is

$$\Phi_B = BxL\sin 60°.$$

The potential difference is due to the induced emf:

$$\mathcal{E} = -\,d\Phi_B/dt = -(dx/dt)BL\sin 60° = -(5.0\text{ m/s})(5 \times 10^{-3}\text{ T})(0.7\text{ m})(0.866) = -1.5 \times 10^{-2}\text{ V}.$$

From Lenz's law, the emf would generate a current to oppose the increase in flux, so it is directed down. The potential difference between the two ends of the bar is

$$\boxed{1.5 \times 10^{-2}\text{ V, with the bottom end at the higher potential}}.$$

26. Assuming that the magnetic force exerted on the rod is small in comparison to the gravitational force, so the motion of the rod is still that of a simple-harmonic motion:

$\theta(t) = \theta_0 \cos(\omega t + \phi_0)$, where $\omega = (L/g)^{1/2}$.

The angular velocity of the rod as a function of time is then

$d\theta/dt = d[\theta_0 \cos(\omega t + \phi_0)]/dt = -\omega\theta_0 \sin(\omega t + \phi_0)$.

The average linear speed of the rod cutting the magnetic field is

$v_{av} = \frac{1}{2}L\,|\,d\theta/dt)\,| = \frac{1}{2}L\omega\theta_0\,|\sin(\omega t + \phi_0)|$.

The induced emf is

$\mathcal{E} = v_{av}BL = \frac{1}{2}BL^2\omega\theta_0\,|\sin(\omega t + \phi_0)|$.

This can be written as a function of θ, as

$|\sin(\omega t + \phi_0)| = [1 - \cos^2(\omega t + \phi_0)]^{1/2} = [1 - (\theta/\theta_0)^2]^{1/2}$; so

$\mathcal{E} = \boxed{\frac{1}{2}BL^{3/2}[g(\theta_0^2 - \theta^2)]^{1/2}}$, where in the last step we noted that $\omega = (L/g)^{1/2}$.

27. (a) $F_B = evB = (1.6 \times 10^{-19}\text{ C})(0.25\text{ m/s})(0.030\text{ T}) = \boxed{1.2 \times 10^{-21}\text{ N}}$.

(b) Let $F_E = eE = eV/d = F_B$ to obtain

$V = F_B\,d/e = evBd/e = vBd = (0.25\text{ m/s})(0.030\text{ T})(0.003\text{ m}) = \boxed{2 \times 10^{-5}\text{ V}}$.

28. When the plate is entering the region between the poles of the electromagnet, the eddy currents will circulate to produce a magnetic field with a direction opposite to the field of the magnet. The magnitudes of the currents will depend on the rate at which the area of the plate within the magnetic field is changing. The magnitudes will increase and then decrease to zero, when the entire plate is within the magnetic field.

When the plate is leaving the region between the poles of the electromagnet, the eddy currents will circulate to produce a magnetic field with a direction the same as the field of the magnet. The magnitudes of the currents will depend on the rate at which the area of the plate within the magnetic field is changing. The magnitudes will increase and then decrease to zero, when the entire plate is outside the magnetic field.

29. We let $t = 0$ when the rod enters the magnetic field. When the rod has traveled a distance $x = vt$, the length of the rod in the magnetic field is $2R\sin\theta$, where

$\cos\theta = (R - vt)/R = 1 - vt/R$ and

$\sin\theta = [1 - (1 - vt/R)^2]^{1/2} = (2vt/R - v^2t^2/R^2)^{1/2}$.

Because $\vec{v}$, $\vec{B}$, and $\vec{L}$ are perpendicular, the motional emf is

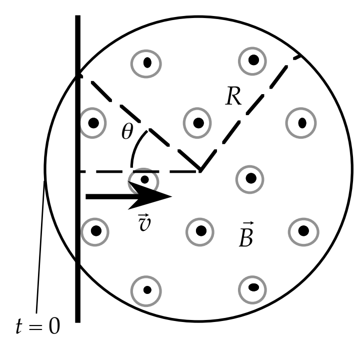
$\mathcal{E} = -BvL = -Bv2R\sin\theta$

$= -Bv2R(2vt/R - v^2t^2/R^2)^{1/2} = -2Bv(2Rvt - v^2t^2)^{1/2}$.

From Lenz's law, we see that the emf will be directed down.

After the rod leaves the field at $t = 2R/v$, the emf will be zero:

$\mathcal{E} = \boxed{-2Bv(2Rvt - v^2t^2)^{1/2}}$ for $0 < t < 2R/v$.



30. To produce the maximum current, the axis of rotation is perpendicular to the field. The flux through the coil is

$\Phi_B = N\vec{B}\cdot\vec{A} = NBA\cos\theta = NBA\cos(\omega t)$.

The induced emf is

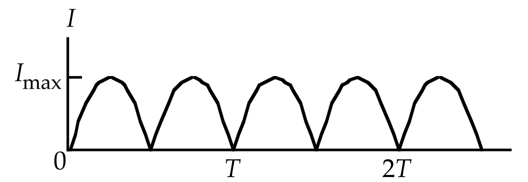
$\mathcal{E} = -d\Phi_B/dt = -NB\,dA/dt = NBA\omega\sin(\omega t)$.

The maximum current in the circuit is produced by the maximum emf:

$I_{max} = \mathcal{E}_{max}/R = NBA\omega/R$, which gives $B = \boxed{I_{max}R/NA\omega}$.

31. (a) Because the split–ring commutator makes the current always positive, we have

$$I = (NBA\omega/R)\,|\sin(\omega t)|\,.$$



(b) The value of $\sin(\omega t)$ is positive for $0 < \omega t < \pi$. We find the average current from the average of $\sin(\omega t)$ for half of a cycle:

$$\left[\sin(\omega t)\right]_{av} = \frac{\int_0^{T/2} \sin(\omega t)\,dt}{\int_0^{T/2} dt} = \frac{\int_{\omega t=0}^{\omega t=\pi} \sin(\omega t)\,d(\omega t)}{\int_{\omega t=0}^{\omega t=\pi} d(\omega t)} = \frac{-\cos(\omega t)\Big|_{\omega t=0}^{\omega t=\pi}}{\pi} = \frac{2}{\pi}.$$

For the average current, we have

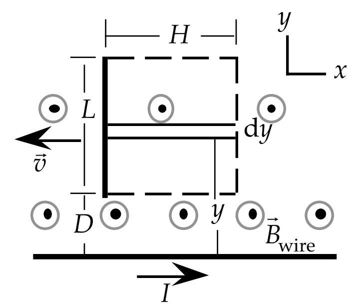
$$I_{av} = (NBA\omega/R)[\sin(\omega t)]_{av} = \boxed{2NBA\omega/R\pi}.$$

(c) For the magnetic field we have $B = \boxed{\pi I_{av}R/2NA\omega}$.

32. The magnetic field is not constant over the area swept out by the rod. We select a differential slice a distance y from the wire and find the magnetic flux through the area by integration:

$$\Phi_B = \iint \vec{B}\cdot d\vec{A} = \int_D^{D+L} \left(\frac{\mu_0 I}{2\pi y}\right) H\,dy$$

$$= \left(\frac{\mu_0 IH}{2\pi}\right)\ln\left(\frac{D+L}{D}\right).$$



The induced emf is

$$\mathcal{E} = -\,d\Phi_B/dt = -\,(\mu_0 I/2\pi)(dH/dt)\,\ln[(D+L)/D]$$

$$= -\,(\mu_0 Iv/2\pi)\,\ln(1 + L/D)$$

$$= -\,[(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A})(0.650\ \mathrm{A})(2.2\ \mathrm{m/s})/2\pi]\times \ln[1 + (45\ \mathrm{cm})/3.5\ \mathrm{cm})]$$

$$= \boxed{-7.5\times 10^{-7}\ \mathrm{V}}.$$

The magnetic flux on the side of the moving rod is into the page. The induced emf in the rod is 7.5×10^{-7} V and the end of the rod closer to the current carrying wire has lower potential.

$\boxed{33}$. We find the current in the loop from

$$I = -\,(d\Phi_B/dt)/R.$$

Because the straight section is perpendicular to both the magnetic field and the velocity, the drag force is

$$F = IBL = (d\Phi_B/dt)BL/R$$

$$= (17\ \mathrm{T\cdot m^2/s})(0.16\ \mathrm{T})(0.12\ \mathrm{m})/(25\ \Omega)$$

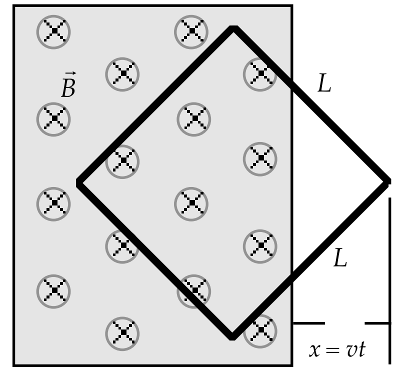
$$= \boxed{1.3\times 10^{-2}\ \mathrm{N}}.$$

The force to keep the loop moving must have the magnitude of the drag force, so the power is

$$P = Fv$$

$$= (1.3\times 10^{-2}\ \mathrm{N})(35\ \mathrm{m/s}) = \boxed{0.46\ \mathrm{W}}.$$

34. The power supplied by the force equals the power dissipated by the current in the resistance of the loop. We take $t = 0$ when the front tip of the loop is leaving the region of the magnetic field. The position of the front tip is $x = vt$.



For $0 < t < L/v\sqrt{2}$, the first half of the loop is leaving the field.
The magnetic flux through the loop is
$\Phi_B = BA = B(L^2 - x^2)$.
The induced emf is
$\mathcal{E} = -\,d\Phi_B/dt = B2x\,(dx/dt) = 2Bxv = 2Bv^2t$.
The power dissipated in the resistance, which must be the power supplied by the external force, is
$P = I^2R = \mathcal{E}^2/R = \boxed{(2Bv^2t)^2/R \quad \text{for } 0 < t < L/v\sqrt{2}}$.
For $L/v\sqrt{2} < t < (2L/v\sqrt{2} = L\sqrt{2}/v)$, the second half of the loop is leaving the field.
The magnetic flux through the loop is
$\Phi_B = BA = B[(2L/\sqrt{2}) - x]^2 = B(L\sqrt{2} - x)^2$.
The induced emf is
$\mathcal{E} = -\,d\Phi_B/dt = -\,B2(L\sqrt{2} - x)\,(-\,dx/dt) = 2B(L\sqrt{2} - x)v = 2Bv(L\sqrt{2} - vt)$.
The power dissipated in the resistance, which must be the power supplied by the external force, is
$P = I^2R = \mathcal{E}^2/R = \boxed{[2Bv(L\sqrt{2} - vt)]^2/R \quad \text{for } L/v\sqrt{2} < t < L\sqrt{2}/v}$.

35. (a) If L is the separation of the rails and D is the distance of the bar from the end of the rails, the magnetic flux through the loop is
$\Phi_B = BA = BLD$.
The induced emf is
$\mathcal{E} = -\,d\Phi_B/dt = -\,BL\,(dD/dt) = -\,BLv$,
which produces a current
$I = \mathcal{E}/R = BLv/R$.
The Lorentz force on this current is
$F = ILB = B^2L^2v/R$, opposite to the direction of the motion.
To maintain the motion, an equal and opposite external force is required:
$F = (0.28\text{ T})^2(0.30\text{ m})^2(0.60\text{ m/s})/(0.050\ \Omega) = \boxed{0.085\text{ N}}$.

(b) The rate of Joule heating in the resistor is
$P = I^2R = (BLv/R)^2R = B^2L^2v^2/R = Fv$
$= (0.085\text{ N})(0.60\text{ m/s}) = \boxed{0.051\text{ W}}$.

36. (a) From Ohm's law
$I = \mathcal{E}/R = 5\text{ V}/2.5\ \Omega = \boxed{2\text{ A}}$, counterclockwise.

(b) $F_B = IBL = (2\text{ A})(0.3\text{ T})(0.12\text{ m}) = \boxed{0.07\text{ N} \quad \text{to the right}}$.

(c) Apply Newton's second law to the movable wire:
$\Sigma F = F_B - F_{\text{resistance}} = m(dv/dt)$;
$IBL - F_{\text{resistance}} = m(dv/dt)$.
Here the current I satisfies
$\mathcal{E} - BLv = IR$, or $I = (\mathcal{E} - BLv)/R$, which we plug in to the equation of motion for the wire:
$[(\mathcal{E} - BLv)/R]BL - F_{\text{resistance}} = m(dv/dt)$; or $a - bv = dv/dt$, where
$a = \mathcal{E}BL/mR - F_{\text{resistance}}/m = (5\text{ V})(0.3\text{ T})(0.12\text{ m})/m(2.5\ \Omega) - (0.02\text{ N})/m$ and
$b = B^2L^2/mR = [(0.3\text{ T})(0.12\text{ m})]^2/m(2.5\ \Omega)$.
The solution to this equation is
$v(t) = (a/b)(1 - e^{-bt}) = \boxed{[(\mathcal{E}BL - F_{\text{resistance}}R)/B^2L^2)]\{1 - \exp[-(B^2L^2/mR)t]\}}$.

37. Let the (constant) terminal speed of the wire be v. Then the induced emf generated in the wire is $\mathcal{E} = BLv$. The resulting current I in the circuit is $I = \mathcal{E}/R = BLv/R$, and the power dissipated due to ohmic heating is

$P_R = I^2R = (BLv/R)^2R = (BLv)^2/R.$

Meanwhile, the power generated by gravity as it pulls the wire of mass m down at speed v is

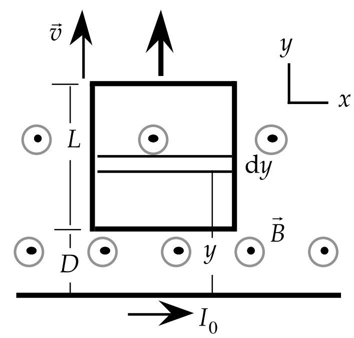
$P_g = F_g v = mgv.$

From the principle of conservation of energy

$P_R = P_g$; $(BLv)^2/R = mgv.$ Thus

$v = \boxed{mgR/(BL)^2}$.

38. (a) The magnetic field of the wire depends on the distance from the wire, $B = (\mu_0/2\pi)I/y$. We select a differential slice a distance y from the wire and find the magnetic flux through the area by integration:



$$\Phi_B = \iint \vec{B}\cdot \mathrm{d}\vec{A} = \int_D^{L+D} \left(\frac{\mu_0 I_0}{2\pi y}\right) L\,\mathrm{d}y$$

$$= \left(\frac{\mu_0 I_0 L}{2\pi}\right)\ln\left(\frac{L+D}{D}\right).$$

The induced emf in the loop is

$\mathcal{E} = -\,\mathrm{d}\Phi_B/\mathrm{d}t = -\,(\mu_0/2\pi)I_0L(-\,L/D^2)(\mathrm{d}D/\mathrm{d}t)/[(L+D)/D]$

$= +\,[(\mu_0/2\pi)I_0L^2/[D(L+D)]v].$

The current in the loop is

$I = \mathcal{E}/R = (\mu_0/2\pi)I_0L^2v/[D(L+D)R] = \mu_0I_0L^2v/[2\pi\, D(L+D)R].$

Because the flux through the loop is decreasing, the current will be counterclockwise.
The force required to pull the loop must balance the net Lorentz force on the loop. The forces on the segments of the loop perpendicular to the wire will be in opposite directions away from the loop, so their net force is zero. The force on the top segment parallel to the wire will be directed up, and the greater force on the bottom segment parallel to the wire will be directed down. The magnitude of the external force is equal to the difference of these two forces:

$F = IL(\mu_0I_0/2\pi D) - IL[(\mu_0I_0/2\pi(L+D)]$

$= [\mu_0I_0L^2v/2\pi\, D(L+D)R]L(\mu_0I_0/2\pi)[1/D - 1/(L+D)] = \boxed{[\mu_0I_0L^2/2\pi D(L+D)]^2(v/R)}$.

(b) The rate at which this force is doing work is

$P = Fv = [(\mu_0I_0L^2/2\pi D(L+D)]^2(v/R)v = \boxed{[\mu_0I_0L^2v/2\pi D(L+D)]^2(1/R)}$.

(c) The Joule heating in the loop is

$P = I^2R = [\mu_0I_0L^2v/2\pi\, D(L+D)R]^2R = [\mu_0I_0L^2v/2\pi D(L+D)]^2/R,$ $\boxed{\text{the same as the answer in (b)}}$.

39. (a) If $v_i < v_t$, the induced emf and the induced current are less, so the drag force is less. The resultant force is directed down; the speed will increase until v_t is reached.

(b) If $v_i > v_t$, the induced emf and the induced current are greater, so the drag force is greater. The resultant force is directed up; the speed will decrease until v_t is reached.

40. Because all elements of the disk have the same angular speed ω, the linear speed of an element at a distance r from the axis is $v = r\omega$. This produces a drag force

$F_d = k'v = k'r\omega.$

In equilibrium, the net torque on the disk is

$\tau_{\text{motor}} - \sum r_iF_{di} = \tau_{\text{motor}} - \sum r_ik'r_i\omega_{\text{eq}} = \tau_{\text{motor}} - k\omega_{\text{eq}} = 0.$

The equilibrium angular velocity is

$\omega_{\text{eq}} = \tau_{\text{motor}}/k,$ which is proportional to the power consumed.

41. We find the current in the loop from

$I = -(\mathrm{d}\Phi_B/\mathrm{d}t)/R.$

The power to pull the cross bar must equal the thermal power generation in the circuit:

$P = I^2R = \boxed{(\mathrm{d}\Phi_B/\mathrm{d}t)^2/R}.$

42. If we ignore the variation of the magnetic field across the loop, the flux through the loop is

$\Phi_B = BA = (\mu_0 I/2\pi a)\,LH.$

The induced emf is

$\mathcal{E} = -\mathrm{d}\Phi_B/\mathrm{d}t = -(\mu_0/2\pi a)LH\,(\mathrm{d}I/\mathrm{d}t) = -(\mu_0/2\pi a)LHI_0(220\pi)[-\sin(220\pi t)].$

The emf is maximum when $\sin(220\pi t) = 1$:

$\mathcal{E}_{\max} = (\mu_0/2\pi a)LHI_0(220\pi) = (\mu_0/4\pi)(440\pi LHI_0/a);$

$1.3 \times 10^{-6}\ \mathrm{V} = (10^{-7}\ \mathrm{T\cdot m/A})(440\pi\ /\mathrm{s})(0.090\ \mathrm{m})(0.008\ \mathrm{m})I_0/(0.020\ \mathrm{m})$, which gives

$I_0 = \boxed{0.26\ \mathrm{A}}.$

43. Because the area vector and the magnetic field are parallel, the flux through the loop is $\Phi_B = BA$. The magnitude of the induced emf is

$\mathcal{E} = \mathrm{d}\Phi_B/\mathrm{d}t = A\,\mathrm{d}B/\mathrm{d}t = A\,\mathrm{d}[B_0(1 - e^{-at})/\mathrm{d}t = \boxed{aA\,B_0\,e^{-at}}.$

44. Because the field increases linearly, the rate of change of the flux, the induced emf and the current are constant. The induced emf is

$\mathcal{E} = \mathrm{d}\Phi_B/\mathrm{d}t = A\,\mathrm{d}B/\mathrm{d}t = A\,\Delta B/\Delta t.$

The current is

$I = \mathcal{E}/R = (A/R)(\Delta B/\Delta t)$

$= [\pi(0.015\ \mathrm{m})^2/(0.15\ \Omega)][(2.7\ \mathrm{T})/(0.05\ \mathrm{s})] = \boxed{0.25\ \mathrm{A}}.$

45. From the cylindrical symmetry, we know that the electric field depends only on r and must be circular. For a circular path just outside the solenoid, the magnetic field through the path is the magnetic field inside the solenoid, $B = \mu_0 nI$, so the flux is

$\Phi_B = BA = B\pi r^2 = \mu_0 nI\pi r^2.$

If we apply Faraday's law, we have

$\int \vec{E}\cdot \mathrm{d}\vec{s} = -\mathrm{d}\Phi_B/\mathrm{d}t;$

$E\int \mathrm{d}s = -\mu_0 n\pi r^2\,(\mathrm{d}I/\mathrm{d}t);$

$E2\pi r = -\mu_0 n\pi r^2 I_0\omega\,[-\sin(\omega t)]$, which gives

$E = \boxed{+(\mu_0 nI_0\omega r/2)\ \sin(\omega t),\ \text{circular}}.$

46. (a) The circumference of each turn in the coil of radius r is $2\pi r$, so the number of turns in the coil made from a wire of length L is $N = L/2\pi r$. The magnetic flux before the current is reversed is BA, where $A = \pi r^2$. After the current is reversed so is the direction of $\vec{B}$, so the new flux is $-BA$. The net change in flux is

$\Delta\Phi_B = -BA - BA = -2\pi r^2 B$, and the corresponding induced emf in the coil is

$\mathcal{E} = -N\,(\mathrm{d}\Phi_B/\mathrm{d}t) = IR$, so the total amount of charge Δq that passes through is

$\Delta q = \int I\,\mathrm{d}t = -(N/R)\int(\mathrm{d}\Phi_B/\mathrm{d}t)\,\mathrm{d}t = -(L/2\pi rR)\int \mathrm{d}\Phi_B$

$= -(L/2\pi rR)\,\Delta\Phi_B = (L/2\pi rR)(2\pi r^2B) = LBr/R$

$= (20\ \mathrm{m})(2.4\ \mathrm{T})(0.043\ \mathrm{m})/5.5\ \Omega = \boxed{0.38\ \mathrm{C}}.$

(b) $I_{\mathrm{av}} = \Delta q/\Delta t = 0.375\ \mathrm{C}/(85 \times 10^{-3}\ \mathrm{s}) = \boxed{4.4\ \mathrm{A}}$, and

$\mathcal{E}_{\mathrm{av}} = I_{\mathrm{av}}\,R = (4.4\ \mathrm{A})(5.5\ \Omega) = \boxed{24\ \mathrm{V}}.$

$\boxed{47}$. (a) The induced emf in the loop due to the change in current in the solenoid is

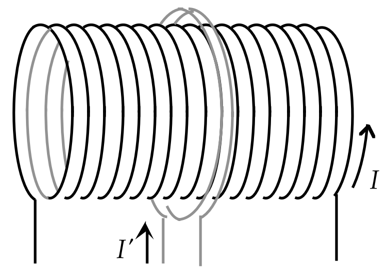
$$\mathcal{E} = -\,\mathrm{d}\Phi_B / \mathrm{d}t = -\,\mathrm{d}(AB)/\mathrm{d}t = -\,A\ (\mathrm{d}B/\mathrm{d}t)$$
$$= -\,(\pi r^2)\ \mathrm{d}(\mu_0 nI)/\mathrm{d}t = -\,(\pi r^2)\ \mu_0 n\ (\mathrm{d}I/\mathrm{d}t) = -\ \pi r^2 \mu_0 n\ (\Delta I/\Delta t)$$
$$= -\,\pi(0.04\ \mathrm{m})^2(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(1800/0.90\ \mathrm{m})(1.2\ \mathrm{A} - 5\ \mathrm{A})/0.3\ \mathrm{s} = 1.6\ \times 10^{-4}\ \mathrm{V}.$$

The induced current in the loop is then

$$I = \mathcal{E}/R = (1.6\ \times 10^{-4}\ \mathrm{V})\ /(6\ \Omega) = \boxed{2.7\ \times 10^{-5}\ \mathrm{A}}.$$

(b) Since the current in the solenoid decreases, the induced current in the loop flows in the same sense as the current in the solenoid.

48. The magnetic field through the loop is $B = \mu_0 nI$ inside the solenoid only, so the flux is



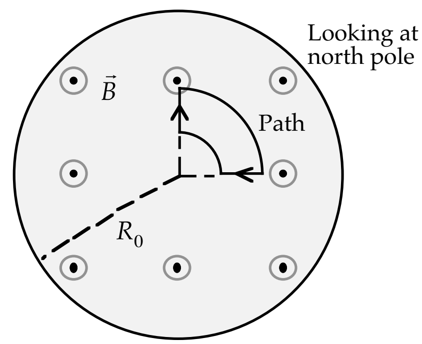
$$\Phi_B = \iint \vec{B} \cdot \mathrm{d}\vec{A} = BA = N\mu_0 nI_0 e^{-t/t_0}\pi r^2.$$

The current induced in the loop is

$$I' = \mathcal{E}/R = -\,(1/R)\ \frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$
$$= -\left(N\mu_0 n\pi r^2 I_0/R\right)\left(-1/t_0\right)e^{-t/t_0} = \left(2\mu_0 n\pi r^2 I_0/Rt_0\right)e^{-t/t_0}.$$

Because the flux through the loop is decreasing, I' will be opposite to the direction shown in the figure.

49. We take the clockwise path as the positive direction.
The magnetic flux through the path is



$$\Phi_B = \vec{B} \cdot \vec{A} = -\,BA$$
$$= -\,B(1/4)[\pi(R/2)^2 - \pi(R/4)^2] = -\,3B\pi R^2/64.$$

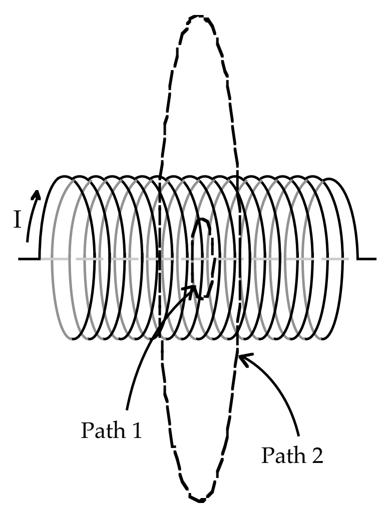
The induced emf is

$$\mathcal{E} = -\,\mathrm{d}\Phi_B/\mathrm{d}t = +\,(3\pi R^2/64)\ \mathrm{d}B/\mathrm{d}t.$$

Because the magnetic field decreases linearly, we have

$$\mathcal{E} = +\,(3\pi R^2/64)\ \Delta B/\Delta t$$
$$= [3\pi(0.08\ \mathrm{m})^2/64][(0.7\ \mathrm{T} - 1.5\ \mathrm{T})/(25 \times 10^{-3}\ \mathrm{s})]$$
$$= \boxed{-0.030\ \mathrm{V}\ \text{(counterclockwise)}}.$$

50. From the cylindrical symmetry, we know that the electric field depends only on r and must be circular.
For path 1, $r < R$, the magnetic field through the path is $B = \mu_0 nI$, so the flux is



$$\Phi_B = BA = B\pi r^2 = \mu_0 nI\pi r^2.$$

If we apply Faraday's law, we have

$$\oint \vec{E} \cdot \mathrm{d}\vec{s} = -\,\mathrm{d}\Phi_B/\mathrm{d}t; \quad E\oint \mathrm{d}s = -\,\mu_0 n\pi r^2\ (\mathrm{d}I/\mathrm{d}t);$$
$E2\pi r = -\,\mu_0 n\pi r^2 I_0\omega\ \cos(\omega t)$, which gives
$$E = -\,(\mu_0 nI_0\omega r/2)\ \cos(\omega t).$$

For path 1, $r = R/2$; we have

$$E = \boxed{-(\mu_0 nI_0\omega R/4)\ \cos(\omega t)\ \text{circular}}.$$

For path 2, $r > R$, the magnetic field through the path is $B = \mu_0 nI$ inside the solenoid only, so the flux is

$$\Phi_B = BA = B\pi R^2 = \mu_0 nI\pi R^2.$$

If we apply Faraday's law, we have

$$\oint \vec{E} \cdot \mathrm{d}\vec{s} = -\,\mathrm{d}\Phi_B/\mathrm{d}t; \quad E\oint \mathrm{d}s = -\,\mu_0 n\pi R^2\ (\mathrm{d}I/\mathrm{d}t);$$
$E2\pi r = -\,\mu_0 n\pi R^2 I_0\omega\ \cos(\omega t)$, which gives $E = -\,(\mu_0 nI_0\omega R^2/2r)\ \cos(\omega t)$.

For path 2, $r = 3R$; we have $E = \boxed{-(\mu_0 nI_0\omega R/6)\ \cos(\omega t)\ \text{circular}}$.

51. (a) We find the maximum current from

$$I_{max} = \mathcal{E}_{max}/R = NAB\omega/R$$
$$= (180 \text{ turns})(6.0 \times 10^{-4}\,\text{m}^2)(0.40 \text{ T})(0.6 \text{ rev/s})(2\pi \text{ rad/rev})/(3\ \Omega)$$
$$= 5.4 \times 10^{-2}\text{ A} = \boxed{54\text{ mA}}.$$

(b) The average power produced is

$$P_{av} = \tfrac{1}{2}P_{max} = \tfrac{1}{2}I_{max}{}^2R$$
$$= \tfrac{1}{2}(5.4 \times 10^{-2}\text{ A})^2(3\ \Omega) = 4.4 \times 10^{-3}\text{ W} = \boxed{4.4\text{ mW}}.$$

52. For a circular coil of radius r, the number of turns from a wire of length L is

$$N = L/2\pi r.$$

The maximum emf is

$$\mathcal{E}_{max} = NAB\omega = (L/2\pi r)(\pi r^2)B(2\pi f) = \pi LrB\,f;$$
$$120\text{ V} = \pi(18\text{ m})r(0.45\text{ T})(300\text{ Hz}), \text{ which gives}$$
$$r = 0.016\text{ m} = \boxed{1.6\text{ cm}}.$$

$\boxed{53}$. If we assume rolling without slipping, the tangential speed of the friction wheel is the tangential speed of the bicycle wheel, which is the linear speed of the bicycle. Thus the angular speed of the friction wheel is $\omega = v/r$, where r is the radius of the friction wheel. We can write the magnetic flux through each coil as

$$\Phi_B = NB_0A\ \cos(\omega t),$$

For the two coils, the induced emf is

$$\mathcal{E} = 2(-\,d\Phi_B/dt) = -\,2NB_0A\omega\ \sin(\omega t).$$

We find the speed of the bicycle from the maximum emf:

$$\mathcal{E}_{max} = 2NB_0A\omega;$$
$$6.4\text{ V} = 2(70)(0.1\text{ T})(8 \times 10^{-4}\text{m}^2)[v/(0.01\text{ m})], \text{ which gives}$$
$$v = \boxed{5.7\text{ m/s}}.$$

54. A spoke of the bicycle wheel will rotate through an angle $\theta = \omega t$, from some arbitrary starting location. The area swept will be

$$A = (\theta/2\pi)\pi R^2 = \tfrac{1}{2}\theta R^2.$$

The magnetic flux through the area is

$$\Phi_B = \iint \vec{B}\cdot d\vec{A} = \tfrac{1}{2}B\theta R^2 = \tfrac{1}{2}B\omega R^2 t.$$

The magnitude of the induced emf is

$$\mathcal{E} = d\Phi_B/dt = \tfrac{1}{2}B\omega R^2$$
$$= \tfrac{1}{2}(0.55\text{ T})(53\text{ rad/s})(0.33\text{ m})^2 = \boxed{1.6\text{ V}}.$$

55. The electric field seen by an observer moving with a velocity $\vec{u}$ is

$$\vec{E}\,' = \vec{E} + (\vec{u} \times \vec{B}) = E\hat{i} + [\vec{u} \times (B\hat{k})].$$

For this electric field to be zero, we have

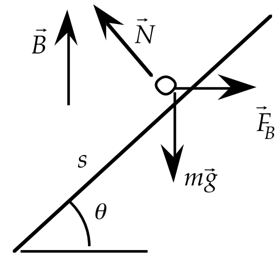
$\vec{u} \times (B\hat{k}) = -\,E\hat{i}$, which gives $\vec{u} = -\,(E/B)\hat{j}$, or $\boxed{\text{a constant speed of } E/B \text{ in the } -y\text{-direction}}$.

If we take $u = 0.1c$ for the nonrelativistic limit, we find the minimum value of B from

$$u = 0.1c = E/B_{min};$$
$$0.1(3 \times 10^8\text{ m/s}) = (10^3\text{ V/m})/B_{min}, \text{ which gives}$$
$$B_{min} = 3 \times 10^{-5}\text{ T}.$$

The range of values of B is $\boxed{B > 3 \times 10^{-5}\text{ T}}$.

56. A side view of the rail and wire is shown in the figure. When the wire is a distance s from the bottom, the magnetic flux through the loop is



$\Phi_B = \iint \vec{B} \cdot d\vec{A} = \vec{B} \cdot \vec{A} = BLs \cos\theta.$

The induced emf is

$\mathcal{E} = -\,d\Phi_B/dt$

$= -(BL\cos\theta)(ds/dt) = -BL(-v)\cos\theta = BLv\cos\theta.$

This produces a current in the wire

$I = \mathcal{E}/R = (BLv\cos\theta)/R$ into the page.

Because the current is perpendicular to the magnetic field, the force on the wire from the magnetic field will be horizontal, as shown, with magnitude

$F_B = ILB = (B^2L^2v\cos\theta)/R.$

For the wire to slide down at a steady speed, the net force must be zero. If we consider the components along the rail, we have

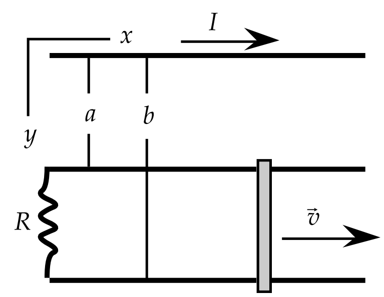
$F_B\cos\theta - mg\sin\theta = 0;$

$[(B^2L^2v\cos\theta)/R]\cos\theta = (B^2L^2v\cos^2\theta)/R = mg\sin\theta;$

$(0.68\ \text{T})^2(1.2\ \text{m})^2v(\cos^2 15°)/(2.0\ \Omega) = (65 \times 10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)\sin 15°$, which gives

$v = \boxed{0.53\ \text{m/s}}.$

57. (a) The magnetic field of the wire depends on the distance from the wire, $B = (\mu_0/2\pi)I/y$. When the moving rod is a distance x from the resistor, we select a differential slice a distance y from the wire and find the magnetic flux through the area by integration:



$$\Phi_B = \iint \vec{B} \cdot d\vec{A} = \int_a^b \left(\frac{\mu_0 I_0}{2\pi y}\right) x\, dy$$

$$= \left(\frac{\mu_0 I_0 x}{2\pi}\right)\ln\left(\frac{b}{a}\right).$$

The induced emf in the rod is

$\mathcal{E} = -\,d\Phi_B/dt = -(\mu_0 I/2\pi)(dx/dt)\ln(b/a)$

$= -(2 \times 10^{-7}\ \text{T}\cdot\text{m/A})(150\ \text{A})(45 \times 10^{-2}\ \text{m/s})\ln[(16\ \text{cm})/(8\ \text{cm})]$

$= \boxed{-9.4 \times 10^{-6}\ \text{V (up)}}.$

(b) To oppose the increase in flux into the page, the induced current in the loop will be counterclockwise, with magnitude

$I_{\text{ind}} = \mathcal{E}/R$

$= (9.4 \times 10^{-6}\ \text{V})/(0.20\ \Omega) = 4.7 \times 10^{-5}\ \text{A} = \boxed{47\ \mu\text{A}}.$

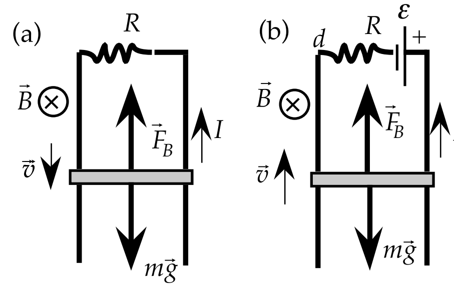
(c) We find the time for the rod to move 100 cm from $\Delta t = \Delta x/v$. The rate at which work is done on the rod must equal the Joule heating. The work done in Δt is

$W = P\,\Delta t = I_{\text{ind}}{}^2 R\,\Delta t$

$= (4.7 \times 10^{-5}\ \text{A})^2(0.20\ \Omega)(1.00\ \text{m})/(45 \times 10^{-2}\ \text{m/s})$

$= \boxed{9.7 \times 10^{-10}\ \text{J}}.$

The work must be done by an $\boxed{\text{external force}}$ applied to the rod. The small value means that the presence of any friction would require much more work by the external force.



58. (a) As the bar falls at constant velocity, the magnetic flux into the loop increases. The induced current is

$I = \mathcal{E}/R = B\ell v/R,$

and will be in the direction shown to create a flux that opposes the increase. There will be an upward magnetic force on this current that balances the weight of the bar:

$mg = I\ell B = (B\ell v/R)\ell B$, which gives

$v = \boxed{mgR/B^2\ell^2}$.

(b) Because the bar is still moving with a constant velocity, the forces must be the same, which means that the current has the same magnitude and direction:

$mg = I\ell B.$

Because the flux is decreasing in the loop, the induced emf must be clockwise. We write a clockwise loop equation for the circuit, starting at point d:

$IR + \mathcal{E} + B\ell v = 0$, which gives $\mathcal{E} = -B\ell v - IR$.

When we substitute for the current and speed, we get

$\mathcal{E} = -B\ell(mgR/B^2\ell^2) - (mg/\ell B)R = \boxed{-2mgR/B\ell,\ \text{polarity opposite from that shown}}$.

59. (a) The magnitude of the magnetic field of the large coil is

$B = \mu_0 NI/2R$, which we can assume is constant over the small ring.

If the coil and the ring are parallel at $t = 0$, the angle between their area vectors is $\theta = \omega t$. The magnetic flux through the ring is

$\Phi_B = \vec{B} \cdot \vec{A}_{\text{ring}} = B\pi r^2 \cos(\omega t) = (\mu_0 NI/2R)\pi r^2 \cos(\omega t).$

The induced emf is

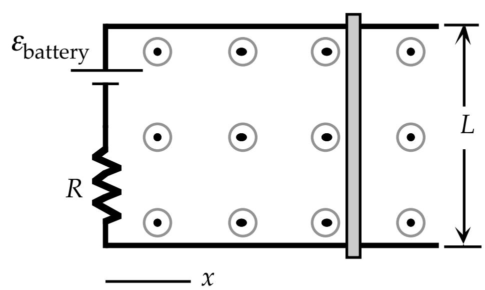
$\mathcal{E} = -\,d\Phi_B/dt$

$= -(\pi r^2 \mu_0 NI/2R)[-\sin(\omega t)]\omega$

$= \boxed{(\pi r^2 \mu_0 NI\omega/2R)\sin(\omega t)}$.

(b) The maximum emf occurs when $\sin\theta = 1$, which gives $\theta = \pi/2 = \boxed{90°}$.

Note that at this angle there is no flux through the small ring, but the rate of change of flux is a maximum.



60. (a) The direction of the current is clockwise, so it is down in the rod. From $\vec{F}_B = I\vec{L} \times \vec{B}$, we see that there is a force toward the battery due to the magnetic field.

$\boxed{\text{The bar accelerates toward the battery}}$.

(b) When the bar of length L is a distance x from the battery, the flux through the circuit is

$\Phi_B = \vec{B} \cdot \vec{A} = BLx.$

The induced emf is

$\mathcal{E}_{\text{ind}} = -\,d\Phi_B/dt = -BL(dx/dt) = -BLv.$

We find the direction of the emf from Lenz's law. Because the flux out of the page is decreasing, the induced current will be counterclockwise to produce an induced field out of the page. The induced emf in the bar will be $\boxed{\text{up}}$.

(c) For the loop circuit, we have

$I = \Sigma\mathcal{E}/R = (\mathcal{E}_{\text{battery}} + \mathcal{E}_{\text{ind}})/R = (\mathcal{E}_{\text{battery}} - BLv)/R$

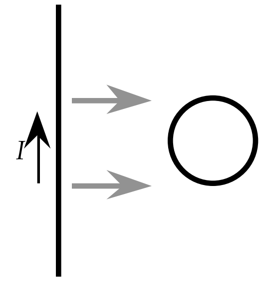
$= [1.50\text{ V} - (0.400\text{ T})(0.200\text{ m})(12.0\text{ m/s})]/(0.30\ \Omega)$

$= \boxed{1.8\text{ A}}$.

61. The magnetic field from the wire is directed into the page with a magnitude that depends on the distance D from the wire:

$B = \mu_0 I/2\pi D$.

As the wire is moved toward the loop, D decreases, so B at the loop will increase. The magnetic flux through the loop, which is directed into the page, will increase. The induced current will produce a magnetic field out of the page, to oppose the increased flux.
Thus the direction of the induced current is $\boxed{\text{counterclockwise}}$.
The induced current in the side of the loop closer to the wire will be in a stronger magnetic field and will experience a greater Lorentz force. Because the force on the side closer to the wire is directed away from the wire, while the force on the side farther from the wire is directed toward the wire, the net force will be $\boxed{\text{away from the wire}}$.



62. The magnetic field produces a force on the moving positive charges toward the point b and on the moving negative charges toward the point a. This accumulation of charge creates an electric field from point b to point a. When equilibrium is reached, in the reference frame moving with the charges there is no electric field:

$\vec{E}' = \vec{E} + \vec{u} \times \vec{B} = 0$, which gives

$u = E/B = V/Bd$

$= (1000 \text{ V})/(2.5 \text{ T})(1 \text{ m})$

$= \boxed{4.0 \times 10^2 \text{ m/s}}$.

63. Because the change in flux through the coil is due to the change in the magnetic field, we have

$\mathcal{E} = -\,d\Phi_B/dt = -\,NA\,dB/dt$.

We find the charge that passes through the coil from

$q = \int I\,dt = \int (\mathcal{E}/R)\,dt = \int (-\,NA/R)\,(dB/dt)\,dt$

$= -\,(NA/R)\int dB = -\,(NA/R)\,\Delta B = -\,(NA/R)(-2B) = 2NAB/R$

$= 2(200 \text{ turns})\pi(4.0 \times 10^{-2} \text{ m})^2(1.4 \text{ T})/(5.6\ \Omega) = \boxed{0.50 \text{ C}}$.

64. (a) In the frame of the coil, the initial flux through the coil is $\Phi_B = BA$. The flux changes due to the reversal of the magnetic field, so we have

$\mathcal{E} = -\,d\Phi_B/dt = -\,NA\,dB/dt$.

We find the charge that passes through the coil from

$q = \int I\,dt = \int (\mathcal{E}/R)\,dt = \int (-\,NA/R)\,(dB/dt)\,dt$

$= -\,(NA/R)\int dB = -\,(NA/R)\,\Delta B = -\,(NA/R)(-\,2B) = 2NAB/R$;

$0.007 \text{ C} = 2(200 \text{ turns})(12 \times 10^{-4} \text{ m}^2)(0.5 \text{ T})/R$, which gives

$R = \boxed{34\ \Omega}$.

(b) The maximum charge through the coil occurs when the change in flux is maximum, which occurs when the coil starts with its plane perpendicular to the magnetic field.
From the analysis of part (a), we have

$q = 2NBA/R$;

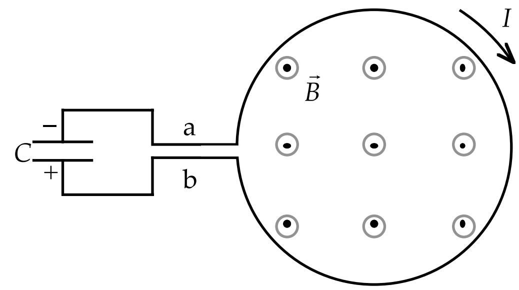
$0.02 \text{ C} = 2(200 \text{ turns})B(12 \times 10^{-4} \text{ m}^2)/(34\ \Omega)$, which gives

$B = \boxed{1.4 \text{ T}}$.

(c) From part (b) the magnetic field is perpendicular to the xy-plane, that is $\boxed{\text{along the } z\text{-axis}}$.

65. Because the flux through the loop is increasing, the induced emf and current will be clockwise and positive charge will accumulate on the lower plate of the capacitor. The magnitude of the induced emf is

$$\mathcal{E} = \mathrm{d}\Phi_B/\mathrm{d}t = A\,\mathrm{d}B/\mathrm{d}t = A\,\mathrm{d}(\alpha t)/\mathrm{d}t = A\alpha.$$



We write a clockwise loop equation for the circuit, starting at point a:

$$A\alpha - IR - Q/C = 0, \text{ with } I = \mathrm{d}Q/\mathrm{d}t.$$

This is the equation for a series RC circuit, which has the solution

$$Q = CA\alpha\left(1 - e^{-t/RC}\right)$$
$$= (15\times10^{-6}\ \mathrm{F})(100\times10^{-4}\ \mathrm{m}^2)(0.03\ \mathrm{T/s})\left[1 - e^{-t/(2\ \Omega)(15\times10^{-6}\ \mathrm{F})}\right]$$
$$= (4.5\times10^{-9}\ \mathrm{C})\left[1 - e^{-t/(3.0\times10^{-5}\ \mathrm{s})}\right], \text{ with lower plate positive.}$$

We find the current in the circuit from

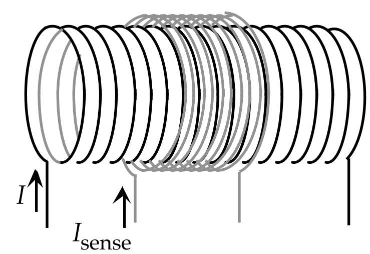
$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{A\alpha}{R}e^{-t/RC}$$
$$= \frac{(100\times10^{-4}\ \mathrm{m}^2)(0.03\ \mathrm{T/s})}{(2\ \Omega)}e^{-t/(2\ \Omega)(15\times10^{-6}\ \mathrm{F})} = (1.5\times10^{-4}\ \mathrm{A})\,e^{-t/(3.0\times10^{-5}\ \mathrm{s})}, \text{ clockwise.}$$

66. (a) The magnetic field through the sense coil is

$$B = \mu_0 nI$$

inside the solenoid only, so the flux is

$$\Phi_B = N_2\iint \vec{B}\cdot \mathrm{d}\vec{A} = N_2\vec{B}\cdot\vec{A}_{\mathrm{solenoid}}$$
$$= N_2\mu_0 nIA_{\mathrm{solenoid}}$$
$$= N_2\mu_0 nI_0A_{\mathrm{solenoid}}\cos(\omega t).$$



The induced emf is

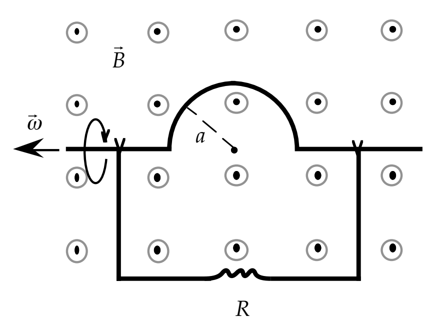
$$\mathcal{E} = -\,\mathrm{d}\Phi_B/\mathrm{d}t$$
$$= -N_2\mu_0 nA_{\mathrm{solenoid}}I_0[-\sin(\omega t)]\omega$$
$$= N_2\mu_0 nA_{\mathrm{solenoid}}I_0\omega\,\sin(\omega t)$$
$$= (10\text{ turns})(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(10^5\text{ turns/m})(10\times10^{-4}\mathrm{m}^2)(10\ \mathrm{A})2\pi(60\ \mathrm{Hz})\,\sin(\omega t)$$
$$= \boxed{4.7\,\sin(120\pi t)\ \mathrm{V}}.$$

(b) The current in the sense coil is

$$I_{\mathrm{sense}} = \mathcal{E}/R = [4.7\sin(120\pi t)\ \mathrm{V}]/(5\ \Omega) = \boxed{0.95\,\sin(120\pi t)\ \mathrm{A}}.$$

67. If we choose $t = 0$ at the position shown, the magnetic flux through the circuit is

$$\Phi_B = \Phi_{B,\mathrm{rectangle}} + \Phi_{B,\mathrm{semicircle}}$$
$$= \Phi_{B,\mathrm{rectangle}} + B(\pi a^2/2)\cos(\omega t).$$



The induced emf is

$$\mathcal{E} = -\,\mathrm{d}\Phi_B/\mathrm{d}t$$
$$= -\,(B\pi a^2/2)[-\sin(\omega t)]\omega = \boxed{(B\pi a^2\omega/2)\,\sin(\omega t)}.$$

We find the average power dissipated from

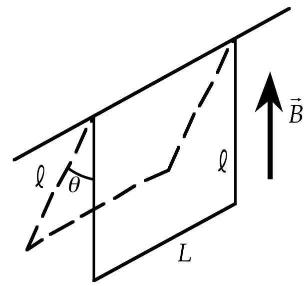
$$I = \mathcal{E}/R;$$
$$P_{\mathrm{av}} = I^2_{\mathrm{av}}R = \mathcal{E}^2_{\mathrm{av}}/R$$
$$= [(B\pi a^2\omega/2)^2/R][\sin^2(\omega t)]_{\mathrm{av}} = [(B\pi a^2\omega/2)^2/R](1/2) = \boxed{B^2\pi^2a^4\omega^2/8R}$$

68. For small oscillations the induced current in the loop will make the system a lightly damped simple pendulum. If the amplitude of the motion changes very slowly, we have

$\theta = \theta_{\max} \sin(\omega t)$, with an angular frequency $\omega = (g/\ell)^{1/2}$.

The energy of the pendulum is

$U = \frac{1}{2} m v_{\max}{}^2 = \frac{1}{2} m g \ell \theta_{\max}{}^2$.



For small angles, the area perpendicular to the magnetic field is

$A_\perp = L\ell \sin\theta \simeq L\ell\theta = L\ell\theta_{\max}\sin(\omega t)$, so the induced emf is

$\mathcal{E} = -B\, \mathrm{d}A_\perp/\mathrm{d}t = -BL\ell\theta_{\max}\omega\cos(\omega t)$.

The average power over a cycle is

$P_{\text{av}} = (\mathcal{E}^2)_{\text{av}}/R = [(BL\ell\theta_{\max}\omega)^2/R][\cos^2(\omega t)]_{\text{av}} = (BL\ell\theta_{\max}\omega)^2/2R$.

For the ratio of the power over a cycle to the energy of the cycle, we have

$P_{\text{av}}/U = [(BL\ell\theta_{\max}\omega)^2/2R]/(\frac{1}{2}mg\ell\theta_{\max}{}^2) = B^2L^2\ell\omega^2/mgR = B^2L^2/mR$

$= (0.12\text{ T})^2(0.10\text{ m})^2/(0.15\text{ kg})(0.030\ \Omega) = \boxed{3.2\times10^{-2}\text{ s}^{-1}}$.

The small value justifies our assumption of light damping, so the amplitude does not change much over a cycle.

69. The train effectively acts as a rod of length L, cutting through the terrestrial magnetic field lines as a speed v. The component of the magnetic field perpendicular to the velocity of the train is $B\cos 30°$, so

$\mathcal{E} = vLB\cos 30°$

$= [(60\text{ mi/h})(1\text{h}/3600\text{ s})(1609\text{ m/mi})]\ (1.5\text{ m})(0.7\times10^{-4}\text{ T})\cos 30°$

$= \boxed{2.4\times10^{-3}\text{ V}}$.

70. As the bar moves forward it generates an induced emf, $\mathcal{E} = BLv$, so a current $I = \mathcal{E}/R = BLv/R$ runs through it, and the bar is therefore subject to a magnetic force of magnitude

$F = BIL = B(BLv/R)L = (BL)^2 v/R$, which is against the direction of motion. The equation of motion of the bar is then

$m\,\mathrm{d}v/\mathrm{d}t = -(BL)^2 v/R$, or $\mathrm{d}v/\mathrm{d}t = -[(BL)^2/mR]\,v$. Rewrite this as

$\mathrm{d}v/v = -[(BL)^2/mR]\,\mathrm{d}t$ and integrate over both sides:

$\int \mathrm{d}v/v = -[(BL)^2/mR]\int\mathrm{d}t$;

$\ln(v/v_0) = -[(BL)^2/mR]\,t$; so

$\boxed{v = v_0 e^{-\alpha t},\ \text{where } \alpha = -(BL)^2/mR}$.

71. (a) The magnetic force provides the centripetal acceleration:

$evB = mv^2/R$, which gives

$v = eBR/m = (1.6\times10^{-19}\text{ C})(10^{-6}\text{ T})(1\text{ m})/(9.1\times10^{-31}\text{ kg}) = \boxed{1.8\times10^5\text{ m/s}}$.

(b) The kinetic energy of the electron is

$E = \frac{1}{2}mv^2 = \frac{1}{2}(9.1\times10^{-31}\text{ kg})(1.8\times10^5\text{ m/s})^2 = \boxed{1.4\times10^{-20}\text{ J}}$.

(c) We find the electric field along the electron's path by applying Faraday's law:

$\int \vec{E}_{\text{field}}\cdot \mathrm{d}\vec{s} = -\mathrm{d}\Phi_B/\mathrm{d}t$;

$E_{\text{field}}\int \mathrm{d}s = E_{\text{field}}2\pi R = -A\,\Delta B/\Delta t = -\pi R^2\,\Delta B/\Delta t$, which gives $E_{\text{field}} = -\frac{1}{2}R\,\Delta B/\Delta t$.

In a time Δt, the electron moves a circular distance $\Delta s = v\,\Delta t$, so the work done by the field, which changes the energy of the electron, is

$W = -eE_{\text{field}}\,\Delta s = -e(-\frac{1}{2}R\,\Delta B/\Delta t)v\,\Delta t = \frac{1}{2}eRv\,\Delta B = \Delta E$.

The fractional energy change is

$\Delta E/E = (\frac{1}{2}eRv\,\Delta B)/(\frac{1}{2}mv^2) = (eR/mv)\,\Delta B$.

If we use the result from part (a) for the speed, we have

$\Delta E/E = [eR/m(eBR/m)]\,\Delta B = \Delta B/B$, which is independent of R and v.

(d) From part (c), we have

$\Delta E/E = \Delta B/B = \boxed{-10\%}$.